Brandles School: Model Calculation Policy

1 Introduction

1.1 The aim of this policy is to outline the school's approach to teaching students how to perform mathematical calculations.

1.2 For any given technique in Mathematics, there are a number of different possible approaches. We believe that adopting a common approach across the Mathematics department, and across the curriculum as a whole, is important because:

1.2.1 we are committed to teaching calculation methods which are rooted in, and reinforce, students' conceptual understanding of Mathematics;

1.2.2 if different teachers employ different methods, this can be confusing for students when they change teachers; and

1.2.3 it strengthens students' understanding and fluency if approaches used within Mathematics are also followed in other subjects.

1.3 All subjects have a role, to a greater or lesser extent, in reinforcing and encouraging students' attainment and confident in Mathematics. This, in turn, can positively benefit students' understanding where Mathematics is used across the curriculum. This policy will outline some important practicalities to enable this to happen.

1.4 This policy applies across the curriculum, wherever Mathematics is used. It is recognised that departments outside Mathematics have many other priorities, and that the policy may be applied with less intensity. However, it is important that the principles and practices herein are applied universally across the school. The Mathematics Department will be pleased to offer support on any aspects of this policy on request.

2 Mental methods of calculation

2.1 Students should be encouraged to use mental methods for simple calculations.

2.2 The majority of students should be able to calculate mentally using the following facts:

2.2.1 Addition and subtraction facts up to 20
2.2.2 Multiplication and division facts involving whole numbers, up to 12×12

However, the speed with which students do this will vary considerably.

2.3 The majority of students will also be able to use mathematical structure to perform mental calculations. Examples include:

$$65 + 28 = 65 + 20 + 8 = 85 + 8 = 93$$

$65 - 28 = 65 - 30 + 2 = 35 + 2 = 37$

$3 \times 52 = 3 \times (50 + 2) = (3 \times 50) + (3 \times 2) = 150 + 6 = 156$

$126 \times 4 = 126 \times 2 \times 2 = 252 \times 2 = 504$

$14 \times 200 = 14 \times 2 \times 100 = 28 \times 100 = 2{,}800$

$78 \div 3 = (60 + 18) \div 3 = (60 \div 3) + (18 \div 3) = 20 + 6 = 26$

$78 \div 6 = 78 \div 2 \div 3 = 39 \div 3 = 13$

2.4 Students should be able to apply these types of mental method to the following:

- 2.4.1 Working out simple percentages (e.g. 20% of £90)
- 2.4.2 Working out simple fractions of quantities (e.g. $\frac{2}{5}$ of 35)
- 2.4.3 Sharing in a simple ratio
- 2.4.4 Calculating areas, volumes and perimeters of simple shapes

2.5 Some students will be able, and should be encouraged, to perform simple calculations involving decimals mentally. Depending on the fluency of the student, examples might include:

$0.4 + 0.8 = 1.2$

$3.2 - 0.02 = 3.18$

$0.3 \times 40 = 3 \times 40 \div 10 = 120 \div 10 = 12$

2.6 It is helpful if teachers discuss with students how they have made a calculation. Any method which can be generalised to give a correct answer is acceptable. However, where an over-complicated method has been used, staff should guide the student towards a more efficient method, if time permits.

3 Written mathematical methods

3.1 Detailed guidance on a variety of common mathematical and calculation methods is contained in Annex A.

3.2 Annex A is not intended to cover every single mathematical technique within the school curriculum. It does, however, include all the techniques which might conceivably be useful in other subjects, in key stages 3, 4 or 5.

3.3 Except where stated otherwise, all methods within Annex A will be taught, at some stage, to nearly all the students in the school. Many of them will actually have been learned in Key Stage 2.

3.4 Members of the Mathematics Department will be pleased to clarify anything in Annex A which is not clear. It will be updated from time to time, in order to reflect changes in mathematical pedagogy, or changes in the curriculum, or in response to requests from members of staff for improved clarity. Staff will be notified of all such updates as they are made.

Students' 'own methods'

3.5 While we will teach 'common methods', students will often use their own methods. This will either be a method which they have been taught elsewhere, or a version they have developed which makes sense to them. So long as the students are confident in their own method, and so long as the method is efficient and will always end up with the right answer, there is no problem with their using it.

3.6 Where students do use their own methods, teachers should check that it is fit for purpose.

3.6.1 If the method is unreliable (for example, it may work in some cases but not others), teachers should employ questioning with the aim of pinpointing misconceptions and leading the student to a more reliable method.

3.6.2 If the method is correct but slow or inefficient, teachers should encourage the use of a more efficient method.

3.7 Where neither of these apply, any attempt to make the student change to the 'standard method' should be used with caution. Where students are comfortable with a particular method, they are often resistant to change, and any pressure to do so could be counterproductive.

4 Students' stages of mathematical development

4.1 When faced with a mathematical calculation or process, some students will be able to understand and implement it with ease. There may be others, at an earlier stage of mathematical development, for whom it is neither as obvious nor as straightforward.

4.2 Students should always be encouraged to understand the method they are using, rather than solely be given a 'recipe' for carrying it out. Understanding a method means it is more likely to be recalled in the future.

4.3 For many of the methods included in Annex A, some indication is given on the likely sequencing of students' understanding. For example, some students prefer a method to be presented pictorially, or even using concrete objects. Such methods are not ends in themselves: they are intended as steps along the way to understanding the final method. The intention will always be for the students, when they are ready, to use the final, abstract method, which will generally be more efficient.

4.4 Teachers of other subjects should be aware that these alternative representations exist, and the role they play in helping students acquire a conceptual understanding of the mathematics they are using. Students should be encouraged to use these representations where they are appropriate to their stage of mathematical development.

5 Good mathematical practices

5.1 Good mathematical practices should be encouraged whenever a calculation is required. The main examples are as follows:

5.1.1 The method of calculation should be shown. Not only does this help to gain credit in examinations, it reinforces the fact that Mathematics is a means of communication: showing working is one facet of this.

5.1.2 Final answers should be checked for reasonableness. If the answer is not 'reasonable', then either the method has been used incorrectly, or a mistake has been made.

5.1.3 The 'equals' sign should be used appropriately. Examples of improper and proper use, for the same problem, is as follows:

Improper usage	**Proper usage**
If $a = 4$:	If $a = 4$:
$2a + 3 = 2 \times 4 = 8 + 3 = \underline{11}$	$2a = 2 \times 4 = 8$
	$2a + 3 = 8 + 3 = \underline{11}$

6 Use of calculator

6.1 Over-reliance on calculators can diminish students' fluency in the use and application of number. Hence, when faced with a calculation:

6.1.1 Students should first seek to use a mental method.

6.1.2 If a mental method is not practical, because of the size or complexity of the numbers, a paper method should be considered.

6.1.3 If neither of these is practical, then a calculator should be used as a last resort.

6.2 Staff should support students in deciding on which method to use, depending on the attainment of the students, and the difficulty of the calculation. For some students, the use of a calculator should be permitted even for seemingly simple calculations, in order that a larger task can be completed successfully. This should, however, be the exception and not the rule.

6.3 In Mathematics lessons, often – though not always – exercises are designed with ease of mental or written calculation in mind. However, in other subjects, where real-life or experimental data are being used, this is often not the case. In such circumstances, use of a calculator is to be encouraged.

6.4 Where particular calculator functions are required (e.g. powers and roots, trigonometrical functions), staff should give specific instruction in their use.

6.5 Students should have their own calculator. Every model of calculator works slightly differently, and it is in the students' interests to become accustomed to how their calculator works. (Brandles Maths Department now has one type of Calculator to be regularly used to increase familiarity and for pupils to be proficient in its use for exams etc.)

6.6 In particular, students should note whether their calculator follows the conventional order of operations (i.e. BIDMAS). Some calculators, including many non-scientific calculators, do not follow BIDMAS, so multi-stage calculations will need to be keyed in differently.

6.7 It is good practice to estimate the answer before using a calculator. It is always necessary to consider whether the answer obtained is sensible.

6.8 Students should always write down the calculation they have done. It is not acceptable for them only to write down the answer.

6.9 Sensible rounding should always be encouraged. While an accuracy of 3 significant figures is often appropriate, in particular cases different degrees of accuracy may be more appropriate. For example:

6.9.1 Where the numbers used in the calculation are themselves approximations, 2 significant figure accuracy is often adequate.

6.9.2 When calculating with money, the final answer should either be given to the nearest pound or the nearest penny (i.e. 2 decimal places). Students should be aware that, for example, if a calculator display of '7.5', in money terms this is '£7.50'.

6.9.3 Angles (for example, in drawing a pie chart) should often be rounded to the nearest degree, or at most to 1 decimal place.

6.9.4 Likewise, percentages should seldom be given to more than 1 decimal place.

6.10 Staff should indicate to students in advance when a forthcoming lesson or unit will require the use of a calculator. Where possible, they should also signpost particular calculations, or activities, where the use of a calculator is permitted.

7 Use of mathematical vocabulary

7.1 Use of the correct vocabulary and terminology is an important element of learning Mathematics.

7.2 Fluent understanding of specialist vocabulary removes barriers and leads to efficient reasoning and problem-solving. Knowledge of the word 'perimeter', for example, will help students access a range of problems in geometry, and enable them to explain solutions more clearly and concisely.

7.3 In Mathematics teaching rooms, the use of word walls is encouraged to familiarise students with relevant vocabulary and to encourage their routine use. This might usefully include words which have the same or similar meaning (for example, 'add', 'plus' and 'sum'). Reminders on vocabulary should also be used occasionally as part of lesson starters.

7.4 Highlighting derivations of words can help students understand and remember their meaning. For example, 'quad' means 'four' (in Latin), 'iso' means 'equal' (in ancient Greek).

7.5 Outside Mathematics lessons, it is helpful for all staff to use agreed vocabulary and terminology, to reinforce students' understanding of the different concepts involved. The department has produced a list of these, which is attached as Annex C.

Annex A

Guidance on common mathematical and calculation methods

Written addition

Efficient method (we aim for all students to understand and use this)	$$\begin{array}{r} 5\,6\,9 \\ +\ \ 8\,7\,6 \\ \hline 1\,4\,4\,5 \\ \hline {\scriptstyle 1\ 1\ \ } \end{array}$$
Other methods that students may use	Partitioning, for example: 569 + 876 = 500 + 60 + 9 + 800 + 70 + 6 = (500 + 800) + (60 + 70) + (9 + 6) = 1300 + 130 + 15 = 1445 This method is also useful when calculating mentally. There are several other mental short-cuts. For example, Change 569 + 876 to 570 + 875 Then change 570 + 875 to 600 + 845 This is then easy to calculate as 1445.
Common errors and misconceptions	
Use of manipulatives and other representations	• Place value counters • Dienes blocks • Number lines (including empty number lines)

Written subtraction

Efficient method (we aim for all students to understand and use this)	$$\begin{array}{r} {}^{1}\cancel{2}\,{}^{10}\cancel{1}\,{}^{12}\cancel{3}\,{}^{1}5 \\ -\ \ 4\,8\,9 \\ \hline 1\,6\,4\,6 \\ \hline \end{array}$$
Other methods that students may use	$$\begin{array}{rrrr} 2000 & 100 & 30 & 5 \\ - & 400 & 80 & 9 \\ \hline 2000 & -300 & -50 & -4 \\ \hline \end{array}$$ 2000 – 300 – 50 – 4 = 1646 There are also several mental short-cuts, for example: Add 11 to each number: 2135 – 489 = 2146 – 500 = 1646
Common errors and misconceptions	Always subtracting the smaller number from the larger number, for example:

<table>
<tr><td></td><td>$$\begin{array}{r} 2\,1\,3\,5 \\ -\quad 4\,7\,9 \\ \hline 2\,3\,4\,4 \\ \hline \end{array}$$</td></tr>
<tr><td>Use of manipulatives and other representations</td><td>

- Place value counters
- Dienes blocks
- Number lines (including empty number lines)

</td></tr>
</table>

'Short' multiplication

<table>
<tr><td>Efficient method (we aim for all students to understand and use this)</td><td>$$\begin{array}{r} 4\,2\,7 \\ \times\qquad 6 \\ \hline 2\,5\,6\,2 \\ \hline {\scriptstyle 1\;\;4\;\;\;} \end{array}$$</td></tr>
<tr><td>Other methods that students may use</td><td>

The grid method below is based on the area of a rectangle, with the larger number partitioned:

<table>
<tr><td></td><td>400</td><td>20</td><td>7</td></tr>
<tr><td>6</td><td>2400</td><td>120</td><td>42</td></tr>
</table>

2400 + 120 + 42
= 2562

This is sometimes laid out in a more regularly-spaced table:

<table>
<tr><td>×</td><td>400</td><td>20</td><td>7</td><td>Total</td></tr>
<tr><td>6</td><td>2400</td><td>120</td><td>42</td><td>2562</td></tr>
</table>

The equivalent written method is:

$$\begin{aligned} 6 \times 427 &= 6 \times (400 + 20 + 7) \\ &= 6 \times 400 + 6 \times 20 + 6 \times 7 \\ &= 2400 + 120 + 42 \\ &= 2562 \end{aligned}$$

This approach is often used by students when working mentally.

</td></tr>
<tr><td>Common errors and misconceptions</td><td>Most mistakes are made through mis-remembering times tables.</td></tr>
<tr><td>Use of manipulatives and other representations</td><td>

- Place value counters
- Tables and grids (as above)
- In simpler cases, Cuisenaire rods on centimetre-squared paper can be used as the edges of a rectangle.

</td></tr>
</table>

'Long' multiplication

<table>
<tr>
<td>Efficient method (we aim for all students to understand and use this)</td>
<td>

$$\begin{array}{cccccc}
 & & 5 & 6 & 1 & 4 \\
 & & \times & & 6 & 7 \\
\hline
3 & 3 & 6 & 8 & 4 & 0 \\
 & {\scriptstyle 3} & & {\scriptstyle 2} & & \\
 & 3 & 9 & 2 & 9 & 8 \\
 & & {\scriptstyle 4} & & {\scriptstyle 2} & \\
\hline
3 & 7 & 6 & 1 & 3 & 8 \\
\hline
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & &
\end{array}$$

</td>
</tr>
<tr>
<td>Other methods that students may use</td>
<td>

An extension of the grid or table used for short multiplication, for example:

×	5000	600	10	4
60	300000	36000	600	240
7	35000	4200	70	28

The 'Gelosia' method is also favoured by some students. (This is also known as the Italian, Chinese or Lattice method.) Its advantage is that it is an easy algorithm to remember; its drawback is that it is difficult for students to understand why it works.

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</tr>
<tr>
<td>Common errors and misconceptions</td>
<td>

The 'efficient' method is seen as long-winded and complicated by many students. The main mistakes made when using it are:

- Mistakes in remembering times tables
- Forgetting to put down the 'zero' when multiplying by the tens digit
- Confusing the tens and units digits

Understanding why the algorithm works can help students avoid these errors.

With the grid method, mistakes can include:

- Mistakes in multiplying the larger numbers (e.g. 60 × 5000 = 30000)
- Not being careful enough in adding the (up to 8) individual figures in columns

</td>
</tr>
<tr>
<td>Use of manipulatives and other representations</td>
<td>

- Place value counters
- Tables and grids (see above)

</td>
</tr>
</table>

‘Short’ division

<table>
<tr>
<td>Efficient method (we aim for all students to understand and use this)</td>
<td>

$$\begin{array}{r|l} & 0\;\;8\;\;7 \quad \text{rem } 1 \\ \cline{2-2} 6 & 5\;{}^{5}2\;{}^{4}3 \end{array}$$

The answer can be written either as ‘87 remainder 1’, or as $87\frac{1}{6}$.
</td>
</tr>
<tr>
<td>Other methods that students may use</td>
<td>

‘Chunking’, using multiplication facts with which the student is comfortable. For example:

$$\begin{array}{lrr} & 523 & \\ 50 \times 6 = 300\text{:} & -\;300 & 50 \\ \cline{2-2} & 223 & \\ 30 \times 6 = 180 & -\;180 & 30 \\ \cline{2-2} & 43 & \\ 7 \times 6 = 42 & -\;\;42 & 7 \\ \cline{2-2} & 1 & \end{array}$$

50 + 30 + 7 = 87, so answer is 87 rem 1
</td>
</tr>
<tr>
<td>Common errors and misconceptions</td>
<td>

Students sometimes forget how to start, especially if (as in the above example), the first digit of the ‘answer’ is a zero.

Other errors often come from misremembering times table facts.
</td>
</tr>
<tr>
<td>Use of manipulatives and other representations</td>
<td>

- Place value counters
- Cuisenaire rods to form the outline of a rectangle with a given area
</td>
</tr>
</table>

‘Long’ division

<table>
<tr><td>Efficient method (we aim for all students to understand and use this)</td><td>

$$\begin{array}{r} 358 \\ 24\,\overline{)\,8610} \\ 72 \\ \hline 141 \\ 120 \\ \hline 210 \\ 192 \\ \hline 18 \end{array}$$

The answer can be written either as ‘358 remainder 18’, $358\frac{18}{24}$, or as $358\frac{3}{4}$.</td></tr>
<tr><td>Other methods that students may use</td><td>‘Chunking’. For the above example:

$$\begin{array}{lrr} & 8610 & \\ 100 \times 24 = 2400: & -\ \ 2400 & 100 \\ \cline{2-2} & 6210 & \\ 100 \times 24 = 2400: & -\ \ 2400 & 100 \\ \cline{2-2} & 3810 & \\ 100 \times 24 = 2400: & -\ \ 2400 & 100 \\ \cline{2-2} & 1410 & \\ 50 \times 24 = 1200: & -\ \ 1200 & 50 \\ \cline{2-2} & 210 & \\ 5 \times 24 = 120: & -\ \ 120 & 5 \\ \cline{2-2} & 90 & \\ 3 \times 24 = 72: & -\ \ 72 & 3 \\ \cline{2-2} & 18 & \end{array}$$

100 + 100 + 100 + 50 + 5 + 3 = 358, so answer is 358 rem 18

Some students will use the ‘short’ division layout, particularly where (as in this example) the divisor is not too large.</td></tr>
<tr><td>Common errors and misconceptions</td><td>The ‘efficient’ method is found to be complex for most students. Using ‘chunking’ can be a lengthy process, with many opportunities for arithmetical errors.</td></tr>
<tr><td>Use of manipulatives and other representations</td><td>

- Place value counters
- For simple examples, Cuisenaire rods to form the outline of a rectangle with a given area
</td></tr>
</table>

Calculating with decimals

Efficient method (we aim for all students to understand and use this)	For many problems, the above methods can be used. – For addition and subtraction, decimal points must be in a vertical line.

	– When multiplying or dividing a decimal by an integer, the decimal point in the answer must be vertically below the decimal point in the original decimal For calculations such as 0.3 × 0.06: $$\frac{3}{10} \times \frac{6}{100} = \frac{18}{1000} = 0.018$$ For calculations such as 0.3 ÷ 0.06: $$\frac{0.3}{0.06} = \frac{300}{6} = 50$$
Other methods that students may use	
Common errors and misconceptions	• Not lining up the decimal points. • (e.g.) 0.4 × 0.3 = 1.2
Use of manipulatives and other representations	• Place value counters • Tables and grids (as for short and long multiplication above)

Multiplying by 10, 100 and other powers of ten

Efficient method (we aim for all students to understand and use this)	When multiplying by 10, all the digits in the number move one place to the left (i.e. they become 10 times larger): 4.57 × 10 = 45.7 0.0078 × 100 = 0.78 1.2 × 1000 = 1200 When dividing, the digits move to the right (i.e. they become 10 times smaller): 4.57 ÷ 10 = 0.457 13 ÷ 1000 = 0.013 5670 ÷ 100 = 5.67
Other methods that students may use	Some students think in terms of moving the decimal point rather than the digits. (It is better to think in terms of place value, as in the efficient methods above.)
Common errors and misconceptions	(e.g.) 4.6 × 10 = 4.60 (i.e. 'just add zero')
Use of manipulatives and other representations	Place value counters

Addition and subtraction of negative numbers

Efficient method (we aim for all students to understand and use this)	Change to an equivalent addition or subtraction, for example:

Calculation	Equivalent	Answer

<table>
<tr><td></td><td>
<table>
<tr><td>4 + (–6)</td><td>4 – 6</td><td>–2</td></tr>
<tr><td>(–2) – (–5)</td><td>(–2) + 5</td><td>3</td></tr>
</table>
Adding a negative number is equivalent to subtracting the positive number.

Subtracting a negative number is equivalent to adding the positive number.
</td></tr>
<tr><td>Other methods that students may use</td><td></td></tr>
<tr><td>Common errors and misconceptions</td><td>“Two minuses make a plus.” This rule applies to multiplication and division, but not addition. For example, students will often write:

$(-3) + (-4) = 7$</td></tr>
<tr><td>Use of manipulatives and other representations</td><td>• Directed number counters, with the understanding that ‘+1’ and ‘–1’ counters form a ‘zero pair’ and cancel each other out.
• Movements up and down a number line</td></tr>
</table>

Multiplication and division of negative numbers

<table>
<tr><td>Efficient method (we aim for all students to understand and use this)</td><td>Use these patterns
<table>
<tr><td>×</td><td>+</td><td>−</td></tr>
<tr><td>+</td><td>+</td><td>−</td></tr>
<tr><td>−</td><td>−</td><td>+</td></tr>
</table>
<table>
<tr><td>÷</td><td>+</td><td>−</td></tr>
<tr><td>+</td><td>+</td><td>−</td></tr>
<tr><td>−</td><td>−</td><td>+</td></tr>
</table>
So, for example: $(-8) \times 3 = -24$
$(-20) \div (-2) = 10$</td></tr>
<tr><td>Other methods that students may use</td><td>Students sometimes remember the rule:

‘same signs positive, different signs negative’

This is simply a way of summarising the two patterns above.

Students may use repeated addition for simple multiplications, for example:

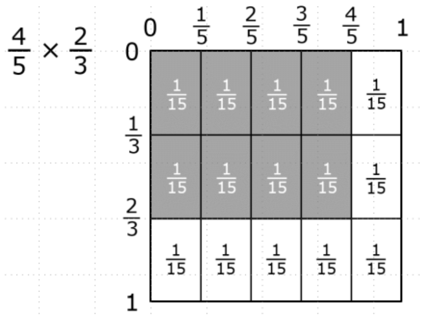
$3 \times (-6) = (-6) + (-6) + (-6) = -18$</td></tr>
<tr><td>Common errors and misconceptions</td><td>For example, $(-4)^2 = -16$, rather than 16</td></tr>
<tr><td>Use of manipulatives and other representations</td><td>• Double number lines to help students understand why (–ve) × (–ve) = +ve</td></tr>
</table>

Addition and subtraction of fractions

Efficient method (we aim for all students to understand and use this)	Find lowest common denominator, and find the equivalent fractions in that denomination, for example: For $\frac{3}{4} - \frac{1}{6}$, the lowest common denominator is 12. $\frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$ For mixed numbers, convert to improper fractions first, for example: $1\frac{1}{4} + 2\frac{3}{5} = \frac{5}{4} + \frac{13}{5}$ These are then added in the same manner as above: $\frac{5}{4} + \frac{13}{5} = \frac{25}{20} + \frac{52}{20} = \frac{77}{20}$ or $3\frac{17}{20}$
Other methods that students may use	Use of common denominators rather than lowest common denominators. This will require cancelling down at the end of the calculation.
Common errors and misconceptions	Simply adding (or subtracting) the numerators and adding (or subtracting) the denominators.
Use of manipulatives and other representations	• Fraction strips • Cuisenaire rods • Bar models

Multiplication of fractions

Efficient method (we aim for all students to understand and use this)	Multiply the numerators and multiply the denominators, for example: $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$ For mixed numbers, convert to improper fractions first, for example: $1\frac{2}{3} \times 1\frac{1}{6} = \frac{5}{3} \times \frac{7}{6} = \frac{5 \times 7}{3 \times 6} = \frac{35}{18}$ or $1\frac{17}{18}$
Other methods that students may use	Some students may simplify before multiplying. The numerator and denominator, even if they are in different fractions, can be divided by a common factor. For example: $\frac{5}{6} \times \frac{9}{16}$: $\frac{5}{{}_{2}\cancel{6}} \times \frac{\cancel{9}^{3}}{16} = \frac{5}{2} \times \frac{3}{16} = \frac{15}{32}$ This is to be recommended where students are fluent in this process.
Common errors and misconceptions	Students sometimes think they need to find common denominators.

	Failing to convert mixed numbers to improper fractions first.
Use of manipulatives and other representations	An area model can be used, as in the example below. 

Division of fractions

Efficient method (we aim for all students to understand and use this)	Dividing by a fraction is the same as multiplying by its reciprocal. $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$ For mixed numbers, convert to improper fractions first, for example: $2\frac{2}{3} \div 3\frac{3}{5} = \frac{8}{3} \div \frac{18}{5} = \frac{8}{3} \times \frac{5}{18}$ $\frac{\cancel{8}^{4}}{3} \times \frac{5}{\cancel{18}_{9}} = \frac{20}{27}$
Other methods that students may use	Students sometimes use the acronym 'KFC' – 'Keep, Flip, Change'. An alternative method is to use a common denominator. $\frac{1}{2} \div \frac{2}{3} = \frac{3}{6} \div \frac{4}{6} = \frac{3}{4}$ (This can be thought of as $\frac{3a}{4a} = \frac{3}{4}$, where $a = \frac{1}{6}$.)
Common errors and misconceptions	Turning the wrong fraction upside-down, or both fractions. (When students use 'KFC', they often forget which fraction to 'keep' or 'flip'.)
Use of manipulatives and other representations	Proportion table. Effectively, this method treats the division as a ratio, and reduces it to the form $x : 1$. $\begin{array}{c\|c\|c\|c} & \frac{1}{2} & \frac{2}{3} & \\ \times 3 \downarrow & \frac{3}{2} & 2 & \downarrow \times 3 \\ \div 2 \downarrow & \frac{3}{4} & 1 & \downarrow \div 2 \end{array}$

Working out a fraction of a quantity

Efficient method (we aim for all students to understand and use this)	$\frac{4}{7}$ of 35: $35 \div 7 = 5$ $5 \times 4 = 20$
Other methods that students may use	Some students would multiply by 4 and then divide by 7. This approach would always work, but the 'efficient' method suggests a better understanding, and will lead to easier arithmetic.
Common errors and misconceptions	Multiplying and dividing by the wrong numbers.
Use of manipulatives and other representations	Bar models

Converting a fraction to a decimal

Efficient method (we aim for all students to understand and use this)	Convert to an equivalent fraction with denominator 10, or 100, or 1000 etc. For example: $\frac{7}{20} = \frac{35}{100} = 0.35$
Other methods that students may use	With a calculator, divide the numerator by the denominator.
Common errors and misconceptions	When using a calculator, dividing the numbers in the wrong order.
Use of manipulatives and other representations	• Bar models to show equivalent fractions • Fraction wall (e.g. to convert fifths to tenths) • Number line to show that 0.1 and $\frac{1}{10}$ have the same value • 100 grid to show that 0.01 and $\frac{1}{100}$ have the same value

Converting a fraction to a percentage

Efficient method (we aim for all students to understand and use this)	Convert to an equivalent fraction with denominator 100, for example: $\frac{14}{25} = \frac{56}{100} = 56\%$ Sometimes, this requires more than one step: $\frac{36}{80} = \frac{9}{20} = \frac{36}{100} = 36\%$
Other methods that students may use	With a calculator, divide the numerator by the denominator. This gives a decimal which can easily be converted to a percentage. (Some students multiply the decimal by 100, but they should be encouraged to do this part mentally.)

Common errors and misconceptions	When using a calculator, dividing the numbers in the wrong order.
Use of manipulatives and other representations	• 100 grid to show that 0.01 and $\frac{1}{100}$ have the same value • Bar model or fraction wall to compare fractions to multiples of 10%

Working out a percentage of a quantity (without a calculator)

Efficient method (we aim for all students to understand and use this)	Work in multiples of 10% or 1%. For example: 30% of 65 = 3 × 6.5 = 19.5 4% of 4200 = 4 × 42 = 168
Other methods that students may use	Students may use 25% or 50%. For example: 35% of 40 = 25% of 40 + 10% of 40 = 10 + 4 = 14 49% of 360 = 50% of 360 – 1% of 360 = 180 – 3.6 = 176.4
Common errors and misconceptions	Because 10% is worked out by dividing by 10, students sometimes assume that, to work out 20%, they need to divide by 20.
Use of manipulatives and other representations	• Bar models for working with multiples of 10%. • 100 grid for working with multiples of 1%

Working out a percentage of a quantity (with a calculator)

Efficient method (we aim for all students to understand and use this)	Using a decimal multiplier. For example, to work out 64% of 85: 0.64 × 85 = 54.4
Other methods that students may use	Dividing by 100 and multiplying by the percentage. So, in the above example: 85 ÷ 100 = 0.85 0.85 × 64 = 54.4 Where possible, students should be encouraged to use multipliers, as it is far quicker. Questions involving compound interest, for example, take ages using the 'slow' method.
Common errors and misconceptions	Dividing and multiplying by the wrong numbers. Using 0.8 for 8% (rather than 0.08). Students often use the non-calculator method (i.e. working in multiples of 10% and 1%). Although this will give the correct answer, it is far slower and should be discouraged in calculator work.

Use of manipulatives and other representations	100 square to emphasise the connection between percentages and decimals

Increasing or decreasing a quantity by a percentage (with a calculator)

Efficient method (we aim for all students to understand and use this)	Using decimal multipliers: To increase £78 by 6%: $1.06 \times 78 = £82.68$ To decrease £230 by 35% $230 \times 0.65 = £149.50$
Other methods that students may use	Work out the percentage and add it on or subtract it. For example: To increase £78 by 6%: $0.06 \times 78 = 4.68$ $78 + 4.68 = £82.68$ While this gives the correct answer, it is inefficient. Students should be encouraged to understand and use the multiplier method.
Common errors and misconceptions	Students not happy about using percentages over 100%. When working out percentage decreases, not subtracting the percentage from 100 (e.g. for a 35% decrease, using a multiplier of 0.35 instead of 0.65).
Use of manipulatives and other representations	• Bar models. These are useful in reinforcing the fact that a 35% decrease leaves 65%. • Double number lines (with percentages on one of the lines)

'Reverse' percentage problems

Efficient method (we aim for all students to understand and use this)	Example: Train fares are increased by 20%. After the increase, a train fare costs £4.20. What was the fare before the increase? $120\% = 4.20$ so $10\% = 4.20 \div 12 = 0.35$ so $100\% = 0.35 \times 100 = £3.50$ (There are variations on this method, for example, dividing by 6 to get 20%, or dividing by 120 to 1%.)
Other methods that students may use	Multiplier for 20% increase is 1.2 $1.2 \times \text{original} = 4.20$ so $\text{original} = 4.20 \div 1.2 = £3.50$
Common errors and misconceptions	Common mistake is to work out the percentage and add it on, or subtract it, from the value given. In the above example, this incorrect method would give: $20\% \text{ of } 4.20 = 0.84$ so $\text{original} = 4.20 - 0.84 = £3.36$

<table>
<tr><td>Use of manipulatives and other representations</td><td>Bar models, particularly for simple percentage changes such as 20%.</td></tr>
</table>

Direct proportion

<table>
<tr><td>Efficient method (we aim for all students to understand and use this)</td><td>There are two methods commonly used.

Example: A recipe for 4 people uses 500 grams of flour. If the recipe is adapted for 6 people, how much flour would be needed?

<u>Unitary method</u>

1 person: 500 ÷ 4 = 125 grams
4 people: 125 × 6 = 750 grams

<u>Multiplier method</u>

Multiplier from 4 to 6 is 6 ÷ 4 = 1.5
500 × 1.5 = 750 grams</td></tr>
<tr><td>Other methods that students may use</td><td>Variations on the unitary method, for example:

2 people: 500 ÷ 2 = 250 grams
4 people: 125 × 3 = 750 grams

Variations on the multiplier method, for example:

$500 \times \frac{6}{4} = 750$</td></tr>
<tr><td>Common errors and misconceptions</td><td>Dividing the wrong way round to obtain the multiplier.

Using an additive method rather than a multiplicative method.</td></tr>
<tr><td>Use of manipulatives and other representations</td><td>

Double number lines
Proportion table (otherwise known as a ratio table, or the 6-box or 4-box method)

<table>
<tr><th></th><th>People</th><th>Flour (g)</th><th></th></tr>
<tr><td></td><td>4</td><td>500</td><td></td></tr>
<tr><td>÷ 2</td><td>2</td><td>250</td><td>÷ 2</td></tr>
<tr><td>× 3</td><td>6</td><td>750</td><td>× 3</td></tr>
</table>
or
<table>
<tr><th></th><th>People</th><th>Flour (g)</th><th></th></tr>
<tr><td></td><td>4</td><td>500</td><td></td></tr>
<tr><td>× 1.5</td><td>6</td><td>750</td><td>× 1.5</td></tr>
</table>
</td></tr>
</table>

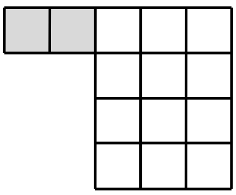
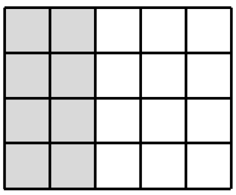
Calculating with ratios

<table>
<tr><td>Efficient method (we aim for all students to understand and use this)</td><td>Example 1: What is the larger share when £168 is shared in the ratio 5 : 2?

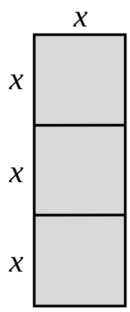
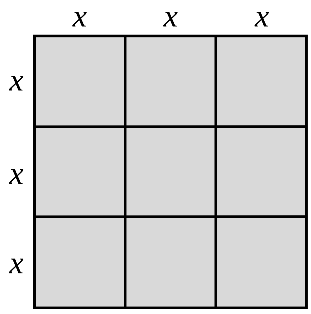
7 shares = 168</td></tr>
</table>

	so 1 share = 168 ÷ 7 = £24 so larger share = 5 × 24 = £120 Example 2: When some money is shared in the ratio 7 : 3, the smaller share is £48. How much money was shared? 3 shares = 48 so 1 share = 48 ÷ 6 = £16 10 shares altogether so total = 10 × 16 = £160
Other methods that students may use	
Common errors and misconceptions	Students can be too quick simply to use the method for Example 1, whatever the problem may actually say.
Use of manipulatives and other representations	• Bar models • Multilink cubes, with different colours to represent the different part of the ratio • Proportion table

Order of operations

Efficient method (we aim for all students to understand and use this)	To evaluate $10 \times 4^2 - 3(12 + 8)$ **B**rackets: $10 \times 4^2 - 3 \times 20$ **I**ndices: $10 \times 16 - 3 \times 20$ **D**ivision: *none* **M**ultiplication: $160 - 60$ **A**ddition: *none* **S**ubtraction: 100 Division and multiplication are interchangeable. Likewise addition and subtraction.
Other methods that students may use	
Common errors and misconceptions	Calculating from left to right, for example: $10 - 3 \times 2 = 7 \times 2 = 14$ (should be $10 - 6 = 4$) $3 \times 2^2 = 6^2 = 36$ (should be $3 \times 4 = 12$)
Use of manipulatives and other representations	Picturing expressions. For example: $2 + 3 \times 4$  $(2 + 3) \times 4$ 

Substituting numbers into an expression

Efficient method (we aim for all students to understand and use this)	Awareness of the conventions of algebraic notation, and use of BIDMAS as above. To evaluate $a(3a + b)^2$ when $a = 2$ and $b = 5$: $2(3 \times 2 + 5)^2$ **B**rackets: 2×11^2 **I**ndices: 2×121 **D**ivision: *none* **M**ultiplication: 242 **A**ddition: *none* **S**ubtraction: *none*
Other methods that students may use	
Common errors and misconceptions	Failure to use the correct order of operations. For example, if $a = 3$: $5 + 2a = 7 \times 3 = 21$ (should be $5 + 6 = 11$) $4a^2 = 12^2 = 144$ (should be $4 \times 9 = 36$)
Use of manipulatives and other representations	Picturing expressions. For example: $3x^2$  $(3x)^2$ 

Solving equations

Efficient method (we aim for all students to understand and use this)	$3(x + 1) = 18 - 2x$ Multiply out brackets: $3x + 3 = 18 - 2x$ Add $2x$ to both sides $5x + 3 = 18$ Subtract 3 from both sides: $5x = 15$ Divide both sides by 5: $x = 3$ This answer can be checked: $3(3 + 1) = 12$ $18 - 2 \times 3 = 12$
Other methods that students may use	Trial and error is generally **not** an acceptable method. For equations where the unknown appears once, a flow chart can be used.
Common errors and misconceptions	Failure to apply the correct inverse operation, for example: $3x = 6 \quad \Rightarrow \quad x = 6 \times 3 \quad \Rightarrow \quad x = 18$

	Getting the order of operations wrong ('undoing' the equation in the wrong order).
Use of manipulatives and other representations	• Bar models are useful for solving equations when all coefficients are positive. • Empty number lines work similarly to bar models. (They work better than bar models when some coefficients are negative.) • Algebra tiles (with negative and positive tiles cancelling out, as with directed number tiles).

Rearranging equations

Efficient method (we aim for all students to understand and use this)	To make y the subject of the equation $x = 2(y + 3p)$: Multiply out brackets: $x = 2y + 6p$ Subtract $6p$ from both sides: $x - 6p = 2y$ Divide both sides by 2: $\frac{x - 6p}{2} = y$
Other methods that students may use	Flow charts. (These don't work if the new subject appears twice.)
Common errors and misconceptions	Failure to apply the correct inverse operation. Getting the order of operations wrong ('undoing' the equation in the wrong order).
Use of manipulatives and other representations	Flow charts

Calculating angles in a pie chart

Efficient method (we aim for all students to understand and use this)	Calculate the number of degrees per item, then multiply by the number of items in each category. <table><tr><td>Red</td><td>6</td></tr><tr><td>Green</td><td>2</td></tr><tr><td>Blue</td><td>3</td></tr><tr><td>Yellow</td><td>4</td></tr></table>Total = 15 360° ÷ 15 = 24° per person <table><tr><td>Red</td><td>6 × 24 = 144°</td></tr><tr><td>Green</td><td>2 × 24 = 48°</td></tr><tr><td>Blue</td><td>3 × 24 = 72°</td></tr><tr><td>Yellow</td><td>4 × 24 = 96°</td></tr></table>
Other methods that students may use	Work out the numbers as percentages first. Then work out each percentage of 360°. Share 360° in a ratio. So, for the above example, share 360 in the ratio $6:2:3:4 = 144:48:72:96$

<table>
<tr><td>Common errors and misconceptions</td><td></td></tr>
<tr><td>Use of manipulatives and other representations</td><td>Proportion table, using a multiplier. For example:
× 24
<table>
<tr><td>6</td><td>144</td></tr>
<tr><td>2</td><td>48</td></tr>
<tr><td>3</td><td>72</td></tr>
<tr><td>4</td><td>96</td></tr>
<tr><td>15</td><td>360</td></tr>
</table>
× 24</td></tr>
</table>

Staff training has taken place on the variety of arithmetical techniques used by pupils in Key Stages 1, 2 and 3. There is an acceptance that pupils are able to tackle the same questions with a variety of methods. These approaches rely on mixing skills, ideas and facts; this is done by pupils drawing on their personal preferences and the particular question. All departments should give every encouragement to pupils using mental techniques but must also ensure that they are guided towards efficient methods and do not attempt convoluted mental techniques when a written or calculator method is required.

Annex B

Reasonable expectations of students

We aim that all students in **Year 7** should:

- have a sense of the size of a number and where it fits in the number system
- know number bonds by heart e.g. tables, doubles and halves
- use what they know by heart to work out answers mentally
- calculate accurately & efficiently using a variety of strategies, both written & mental
- recognise when AND when not to use a calculator; using it efficiently if needs be
- make sense of number problems, including non-routine problems, and recognise the operations needed to solve them
- explain their methods and reasoning using correct mathematical terms
- judge whether their answers are reasonable, and have strategies for checking
- suggest suitable units for measuring
- make sensible estimates for measurements
- explain and interpret graphs, diagrams, charts and tables
- use the numbers in graphs, diagrams, charts and tables to predict.

We aim that all students in **Year 9** should:

- have a sense of the size of a number and where it fits into the number system;
- recall mathematical facts confidently;
- calculate accurately and efficiently, both mentally and with pencil and paper, drawing on a range of calculation strategies;
- use proportional reasoning to simplify and solve problems;
- use calculators and other ICT resources appropriately and effectively to solve mathematical problems, and select from the display the number of figures appropriate to the context of a calculation;
- use simple formulae and substitute numbers in them;
- measure and estimate measurements, choosing suitable units and reading numbers correctly from a range of meters, dials and scales;
- calculate simple perimeters, areas and volumes, recognising the degree of accuracy that can be achieved;
- understand and use measures of time and speed, and rates such as £ per hour or miles per litre;
- draw plane figures to given specifications and appreciate the concept of scale in geometrical drawings and maps;
- understand the difference between the mean, median and mode and the purpose for which each is used;
- collect data, discrete and continuous, and draw, interpret and predict from graphs, diagrams, charts and tables;
- have some understanding of the measurement of probability and risk;
- explain their methods, reasoning and conclusions, using correct mathematical terms;
- judge the reasonableness of solutions and check them when necessary;
- give their results to a degree of accuracy appropriate to the context.

Annex C

Correct use of mathematical terminology

The words on this list are those whose exact mathematical meaning needs to be better known. All teachers should endeavour to use these with precision, and encourage students to do the same.

Further suggestions for this list are welcome.

Sum
: Specifically used to denote addition. For example, the sum of 34 and 98 is 34 + 98 = 132.

  The word is often incorrectly used to mean any calculation.

Product
: Specifically used to denote multiplication. For example, the product of 8 and 5 is 8 × 5 = 40.

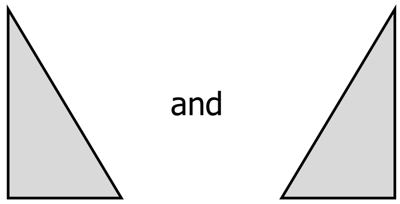
Multiply
: This word should be used in preference to ‘times’
  For example, ‘5 multiplied by 17’ rather than ‘5 times 17’

Minus
: A binary operation, meaning it acts on two numbers. It should only be used in this context, for example, ‘9 minus 4 equals 5’.

Negative
: A unary operation: it acts on one number only. (–6 is the negative of 6.) The number ‘–6’ should be read as ‘negative 6’, not ‘minus 6’.

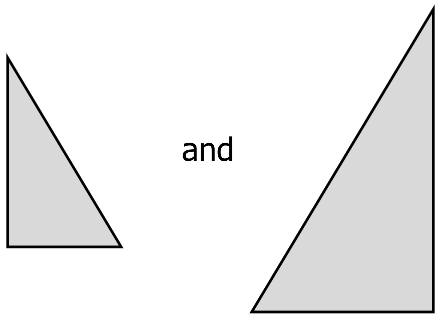
Congruent
: Refers to shapes that are exactly the same shape and size.

  The two shapes may be reflections or rotations of each other, for example:



Similar
: Refers to shapes that are the same shapes but not the same size. Each shape is an enlargement of the other.

  The shapes may be in different orientations, or reflected, for example:



Reading decimals	The number 3.14 should be read as 'three point one four', not 'three point fourteen'.
Calculate	This should not be taken to mean that a calculator should be used. People can calculate mentally, or using paper methods
Estimate	Means an accurate answer is not required.
	The word might be used to refer to a measurement (e.g. 'Estimate the height of the building').
	It can also be used to refer to the answer to a calculation. For example, to estimate the answer to 318.93×78.3:

$$318.93 \times 78.3 \approx 300 \times 80 = 24{,}000$$